MODELLING OF LOVE-TYPE WAVES IN AN ELASTIC LAYER SANDWICHED BETWEEN VISCOUS LIQUID HALF SPACE AND SIZE DEPENDENT COUPLE STRESS SUBSTRATE

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Dispersion curves employed for designing Love wave based liquid sensing devices may provide more accurate information if due consideration is given to parameters describing microstructural behavior of the substrate. The present study involves mathematical modelling of Love waves propagating in a hybrid structure consisting of an elastic layer in the middle overlying a size dependent substrate, loaded with a viscous liquid (Newtonian) half space. Numerical computations are carried out to graphically demonstrate the effects of various parameters: characteristic length of the substrate, thickness of the elastic layer, viscosity and density of the overlying viscous liquid (Newtonian) on dispersion characteristics.

Keywords: Love-type waves, couple stress, viscous liquid, viscosity, sensors

1. Introduction

Due to shear horizontal particle displacements, Love-type wave based devices can operate effectively both in liquid and gas environments. So these devices are being used as viscosity sensors, bio sensors and chemical sensors (Vellekoop, 1998; Wang et al., 2008). The application of surface acoustic waves (SAW) for nondestructive diagnostics of layered media was presented by Kuznetsov (2010). For detection of analytes in liquid, SAW Love mode sensors with high sensitivity were used (Rocha-Gaso et al., 2009). To enhance the performance of conventional SAW devices based on such periodic wave guiding layers, several structure models were reviewed (Xu and Yuan, 2018). These Love-type wave based devices consist of an elastic layer bonded to an elastic substrate. Due to the difference in mechanical properties of the substrate, the layer acoustic energy gets entrapped near the surface causing penetration of the wave in the substrate. Due to in-plane behavior of the Love wave propagation, a larger part of the wave is confined in the guiding layer only, thus enhancing sensitivity of the device. Many researchers have studied the influence of an inviscid liquid on acoustic waves propagating in elastic materials (Sharma and Kumar, 2018; Kim, 1992; Guo and Sun, 2008). Kielczynski et al. (2012) studied the effect of a viscous liquid loading on the Love wave propagation. Baroi et al. (2018) studied propagation of polarized shear horizontal waves in the viscous liquid layer resting over a porous piezoelectric half-space. Vikstrom and Voinova (2016) investigated surface acoustic waves with horizontal polarization (SH-SAWs) propagating in a three-layer system consisting of an elastic substrate and two viscoelastic overlayers. The effect of gravity, heterogeneity and internal friction on propagation of SH-waves (horizontally polarised shear waves) in a viscoelastic layer over a half-space was studied by Sahu *et al.* (2014).

For designing of Love-type wave based devices the obtained dispersion relation is very important. This relation provides phase velocity of the wave in terms of guiding layer thickness and physical properties of the substrate and layer. Different researchers have provided dispersion curves for one or more layers on a substrate. However, ignoring the size dependence of the substrate may lead to less valid results. Therefore, for designing more efficient Love-type wave based devices, a general dispersion relation is needed by considering the size dependent properties of the substrate. Models capturing the size effects have been provided by various researchers (Gunther, 1958; Toupin 1962; Mindlin and Tiersten, 1962; Koiter, 1964; Eringen, 1968; Nowacki, 1974). All these models proposed an additional material length scale parameter to address the size effects. Various reformulations have been applied to these elastic theories. The consistent couple stress theory given by Hadjesfandiari and Dargush (2011) is attractive because it is simple, relatively less complex due to a lesser number of length scale parameters. This theory also claims to resolve all the inconsistencies in the Mindlin and Tiersten couple stress theory (Mindlin and Tiersten, 1962) by discovering the skew symmetric character of the couple stress coefficient and two Lame parameters λ and μ . Further, η depends upon the characteristic length parameter *l*. Using this model, various problems of wave propagation phenomena have been discussed (Sharma and Kumar, 2014, 2017; Ghodrati *et al.*, 2017).

Considering the merits of the size dependent model over the classical model, here we intend to study the effect of a viscous fluid loading on the Love wave propagation in a layered structure with a substrate exhibiting microstructural properties. The effect of characteristic length parameter l, viscosity parameter η_1 , density parameter ρ_1 and thickness H of the sandwiched layer are observed and plotted. The results obtained in this paper can be used significantly for designing Love-type wave based devices in liquid-phase environments – a situation typical of biosensors.

2. Formulation and solution of the problem

To model the present problem, we have considered a cartesian coordinate system in such a way that the Love wave is propagating along x-axis and z-axis is considered positive in the vertically downward direction. The wave guide surface is at z = -H and is loaded with a viscous liquid (Newtonian) of viscosity η_1 and density ρ_1 . The basic configuration supporting propagation of the Love waves, consists of a finite layer which is deposited on the semi-infinite substrate, and the shear wave velocity in the layer is less than that of the substrate. z = 0 is the common interface in which the Love wave propagates. The hybrid structure consists of an elastic surface lying over a couple stress half space with microstructural properties, characterized by an additional material length scale parameter, characteristic length l, loaded with a viscous liquid (Newtonian). The thickness of the elastic layer is H. The considered problem is two dimensional, having no variation along the y-axis.

Let $u_i^e = (u_1^e, u_2^e, u_3^e)$ and $u_i^c = (u_1^c, u_2^c, u_3^c)$ are the mechanical displacement components in the middle elastic layer and the lower couple-stress half space, respectively, obtained due to propagation of the Love wave. As the Love wave is propagating along the direction of x-axis, it causes a displacement in the y-direction only. We shall assume that

$$u_1^e = 0 u_2^e = u_2^e(x, z, t) u_3^e = 0 (2.1)$$
$$u_1^c = 0 u_2^c = u_2^c(x, z, t) u_3^c = 0$$

2.1. Viscous liquid region (z < -H)

The Navier-Stokes equation for a viscous liquid (Newtonian) half space (z < -H) with velocity field v_2 (of the SH acoustic wave) is given by

$$\frac{\partial v_2}{\partial t} - \frac{\eta_1}{\rho_1} \left(\frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial z^2} \right) = 0 \tag{2.2}$$

where η_1 is viscosity and ρ_1 is density of the viscous liquid (Newtonian).



Fig. 1. Geometry of the problem

We assume the solution to (2.2) of the velocity field v_2 in the viscous liquid be

$$v_2(x, z, t) = V(z) \exp[i(kx - \omega t)]$$

$$(2.3)$$

where ω is the angular frequency and k is the angular wave number.

Substitution of (2.3) into (2.2) gives

$$V''(z) - \left(k^2 - \frac{i\omega\rho_1}{\eta_1}\right)V(z) = 0 \qquad V''(z) - \gamma^2 V(z) = 0$$
(2.4)

where

$$\gamma = \sqrt{k^2 - \frac{\mathrm{i}\omega\rho_1}{\eta_1}}$$
 $\operatorname{Re}(\gamma) > 0$

It should be Solving (2.4), we get

$$V(z) = A_1 \exp(\gamma z) + B_1 \exp(-\gamma z)$$
(2.5)

where A_1 and B_1 are arbitrary constants.

With increasing distance from the wave guide surface $z \to -\infty$, the amplitude of the Love wave decays to zero. The condition $\operatorname{Re}(\gamma) > 0$ assures this phenomenon.

From (2.3) and (2.5), the final solution for the velocity component in the viscous liquid is

$$v_2(x, z, t) = A_1 \exp(\gamma z) \exp[i(kx - \omega t)]$$
(2.6)

With the help of (2.6), the only shear stress component is given by

$$\tau_{yz}^{l} = \eta_1 \frac{\partial v_2}{\partial z} = A_1 \eta_1 \gamma \exp(\gamma z) \exp[i(kx - \omega t)]$$
(2.7)

2.2. Elastic surface layer (-H < z < 0)

The equation of motion in the absence of body forces for the elastic surface layer (-H < z < 0) is given by

$$\frac{1}{c_1^2}\frac{\partial^2 u_2^e}{\partial t^2} = \frac{\partial^2 u_2^e}{\partial x^2} + \frac{\partial^2 u_2^e}{\partial z^2}$$
(2.8)

where $c_1 = \sqrt{\mu_2/\rho_2}$ is bulk shear wave velocity in the layer.

We assume the solution to (2.8) of the mechanical displacement u_2^e of the Love wave in the elastic surface layer as

 $u_2^e(x, z, t) = f(z) \exp[i(kx - \omega t)]$ (2.9)

Substitution of (2.9) into (2.8) gives

$$f''(z) + p^2 f(z) = 0 (2.10)$$

where

$$p^2 = \frac{\omega^2}{c_1^2} - k^2$$

Solution to this differential equation (2.10) is substituted into equation (2.9), then we get the displacement component as

$$u_2^e(x, z, t) = [A_2 \cos(pz) + A_3 \sin(pz)] \exp[i(kx - \omega t)]$$
(2.11)

where A_2 and A_3 are arbitrary constants.

The shear stress component will be given by

$$\tau_{yz}^{e} = \mu_2 \frac{\partial u_2^{e}}{\partial z} = [-A_2 \sin(pz) + A_3 \cos(pz)] \mu_2 p \exp[i(kx - \omega t)]$$
(2.12)

2.3. Couple stress layer (z > 0)

The basic governing equation of motion and the constitutive relation of couple stress theory for an isotropic material in the absence of body forces (Hadjesfandiari and Dargush, 2011) are given by

$$(\lambda + \mu + \eta \nabla^2) \nabla (\nabla \cdot \mathbf{u}_i^c) - (\mu - \eta \nabla^2) \nabla^2 \mathbf{u}_i^c = \rho \frac{\partial^2 \mathbf{u}_i^c}{\partial t^2}$$

$$\tau_{ji} = \lambda u_{k,k}^c \delta_{ij} + \mu (u_{i,j}^c + u_{j,i}^c) - \eta \nabla^2 (u_{i,j}^c - u_{j,i}^c)$$

$$\mu_{ji} = 4\eta (\omega_{i,j} - \omega_{j,i})$$
(2.13)

where

$$\omega_i = \frac{1}{2} \epsilon_{ijk} u_{k,j}^c$$

Here u_i^c are the displacement components. λ and μ are Lame constants, $\eta = \mu l^2$ is the couple stress coefficient, l is characteristic length, ρ is density of the material. τ_{ji} is the symmetric stress tensor, δ_{ij} is Kronecker's delta and ϵ_{ijk} is the permutation tensor, and i, j, k = 1, 2, 3.

Let us assume that $\mathbf{u}_i^c = [0, u_2^c, 0]$ is the displacement vector and $\partial/\partial y \equiv 0$.

Imposing the above said conditions, equation of motion (2.13) becomes

$$\frac{\partial^2 u_2^c}{\partial x^2} + \frac{\partial^2 u_2^c}{\partial z^2} - l^2 \Big(\frac{\partial^4 u_2^c}{\partial x^4} + 2 \frac{\partial^4 u_2^c}{\partial x^2 \partial z^2} + \frac{\partial^4 u_2^c}{\partial z^4} \Big) = \frac{1}{c_2^2} \frac{\partial^2 u_2^c}{\partial t^2}$$
(2.14)

where $c_2^2 = \mu/\rho$.

Let us assume the solution to above equation (2.14) as

$$u_2^c(x, z, t) = g(z) \exp[i(kx - \omega t)]$$
 (2.15)

where $\omega = kc$ is the angular frequency, k is the wave number and c is the phase velocity.

Equation (2.14) reduces to

$$\frac{d^4g(z)}{dz^4} - S\frac{d^2g(z)}{dz^2} + Pg(z) = 0$$
(2.16)

where

$$S = \frac{1}{l^2} + 2k^2 \qquad P = k^4 + \frac{k^2}{l^2} - \frac{\omega^2}{l^2 c_2^2}$$

Solution to the above differential equation becomes

$$f(z) = A_4 \exp(-\alpha z) + A_5 \exp(-\beta z)$$
 (2.17)

where A_4 and A_5 are arbitrary constants

$$\alpha = \sqrt{\frac{S + \sqrt{S^2 - 4P}}{2}} \qquad \beta = \sqrt{\frac{S - \sqrt{S^2 - 4P}}{2}} \tag{2.18}$$

Hence, the displacement component is given by

$$u_2^c(x, z, t) = [A_4 \exp(-\alpha z) + A_5 \exp(-\beta z)] \exp[i(kx - \omega t)]$$
(2.19)

The shear stress component is given by

$$\tau_{yz}^c = \mu \frac{\partial u_2^c}{\partial z} + \eta \Big(\frac{\partial^3 u_2^c}{\partial x^2 \partial z} + \frac{\partial^3 u_2^c}{\partial z^3} \Big)$$
(2.20)

The couple stress component is given by

$$\mu_{xz}^c = 4\eta \Big(\frac{\partial \omega_1}{\partial z} - \frac{\partial \omega_3}{\partial x}\Big) \tag{2.21}$$

Using (2.19) in above equations (2.20) and (2.21), we get

$$\tau_{yz}^c = [A_4 \alpha p_1 \exp(-\alpha z) + A_5 \beta q_1 \exp(-\beta z)] \exp[i(kx - \omega t)]$$
(2.22)

where

$$p_1 = -\mu + k^2 \eta - \alpha^2 \eta$$
 $q_1 = -\mu + k^2 \eta - \beta^2 \eta$

and

$$\mu_{xz}^{c} = -2\eta [A_4(\alpha^2 - k^2) \exp(-\alpha z) + A_5(\beta^2 - k^2) \exp(-\beta z)] \exp[i(kx - \omega t)]$$
(2.23)

3. Boundary conditions

The following are boundary conditions to be satisfied in the considered model for propagation of the Love wave.

• Velocity components should be continuous at the interfacial surface

$$\frac{\partial u_2^e}{\partial t} = v_2 \qquad \text{at} \quad z = -H \tag{3.1}$$

• Continuity of the displacement components at the interface

$$u_2^e = u_2^c \qquad \text{at} \quad z = 0 \tag{3.2}$$

• The magnitude of the shear component of the stress tensor should be equal at the interface

$$\begin{aligned} \tau_{yz}^l &= \tau_{yz}^e & \text{at} \quad z = -H \\ \tau_{yz}^c &= \tau_{yz}^e & \text{at} \quad z = 0 \end{aligned} \tag{3.3}$$

• Couple stress of the substrate should vanish at the interface

$$\mu_{xz}^c = 0 \qquad \text{at} \quad z = 0 \tag{3.4}$$

4. Derivation of secular equation

Using the above mentioned boundary conditions from (3.1) to (3.4), we obtain the following equations in terms of five unknown coefficients A_1 , A_2 , A_3 , A_4 and A_5 as

$$\exp(-\gamma H)A_{1} + i\omega\cos(pH)A_{2} - i\omega\sin(pH)A_{3} = 0$$

$$A_{2} - A_{4} - A_{5} = 0$$

$$\eta_{1}\gamma\exp(-\gamma H)A_{1} - \mu_{2}p\sin(pH)A_{2} - \mu_{2}p\cos(pH)A_{3} = 0$$

$$\mu_{2}pA_{3} - \alpha p_{1}A_{4} - \beta q_{1}A_{5} = 0$$

$$(\alpha^{2} - k^{2})A_{4} + (\beta^{2} - k^{2})A_{5} = 0$$
(4.1)

To obtain a non-trivial solution, the determinant of the coefficients of the unknowns A_1 , A_2 , A_3 , A_4 and A_5 should vanish. By solving the determinant, we get the following dispersion equation for the Love wave in a sandwiched elastic layer loaded with a viscous liquid lying over a couple stress substrate

$$\mu_2 p \cos(pH) [\beta q_1(\alpha^2 - k^2) - \alpha p_1(\beta^2 - k^2)] - \mu_2^2 p^2 \sin(pH)(\beta^2 - \alpha^2) + \eta_1 \gamma i \omega \{ \sin(pH) [\alpha p_1(\beta^2 - k^2) - \beta q_1(\alpha^2 - k^2)] - \mu_2 p \cos(pH)(\beta^2 - \alpha^2) \} = 0$$
(4.2)

The real part of (4.2) gives the dispersion equation, and the imaginary part gives the damping equation associated with the Love surface wave propagation. After separating the real and imaginary parts of (4.2), we get the dispersion and damping equations as

$$(R_3 + \gamma_2 R_5)R_1 - (R_4 - \gamma_2 R_6)R_2 = 0$$

$$\gamma_1(R_5 R_1 + R_6 R_2) = 0$$
(4.3)

where

$$R_{1} = \beta q_{1}(\alpha^{2} - k^{2}) - \alpha p_{1}(\beta^{2} - k^{2}) \qquad R_{2} = \mu_{2} p(\beta^{2} - \alpha^{2})$$

$$R_{3} = \mu_{2} p \cos(pH) \qquad R_{4} = \mu_{2} p \sin(pH) \qquad R_{5} = \eta_{1} \omega \sin(pH)$$

$$R_{6} = \eta_{1} \omega \cos(pH) \qquad T = \frac{\omega \rho_{1}}{\eta_{1}} \qquad \gamma = \gamma_{1} + i\gamma_{2}$$

$$\gamma_{1} = \sqrt{\frac{T}{2}} + \frac{k^{2}}{2\sqrt{2T}} \qquad \gamma_{2} = -\sqrt{\frac{T}{2}} + \frac{k^{2}}{2\sqrt{2T}}$$

5. Numerical results and discussion

Numerical simulations have been performed to analyze the theoretical results obtained in the previous Sections. Following data has been taken into account for the graphical illustration:

- (i) For the viscous liquid layer: (Kielczynski *et al.*, 2012) $\mu_1 = 50 \text{ Pas}, \ \rho_1 = 1.10^3 \text{ kg/m}^3.$
- (ii) For the elastic layer: (Kielczynski *et al.*, 2012) The thickness of the layer is assumed to be 0.4 mm, $\mu_2 = 3.91 \cdot 10^{10} \,\mathrm{N/m^2}, \ \rho_2 = 8.9 \cdot 10^3 \,\mathrm{kg/m^3}, \ c_1 = \sqrt{\mu_2/\rho_2} = 3206.5 \,\mathrm{m/s}.$
- (iii) For the couple stress layer: (Sharma and Kumar, 2017) $\mu = 30.5 \cdot 10^9 \,\text{N/m}^2, \ \rho = 2717 \,\text{kg/m}^3.$

The graphs depicting the microstructural effects of the substrate, viscosity and density effects of fluid loading and thickness effects of the sandwiched elastic layer are shown in Figs. 2-5 for the dimensionless phase velocity c/c_1 versus the dimensionless wave number kH. One common feature among all profiles is that the phase velocity decreases with an increase in the wave number.

5.1. Effect of microstructure

To better observe the dependence of the material length scale parameter over the Love wave propagation, the dispersion curves are compared for different values of characteristic lengths as shown in Fig. 2. It is pointed out by the researchers that it is not possible to find the exact



Fig. 2. Variation of the dimensionless phase velocity c/c_1 against the dimensionless wave number kH for different values of the characteristic length l

value of the characteristic length. It is of the order of cell size of the considered material. The profile has been plotted by taking the characteristic length to be of the order of 10^{-4} . We have considered the viscosity and density of the viscous liquid (Newtonian) as $\eta_1 = 50$ Pas

and $\rho_1 = 1000 \text{ kg/m}^3$, respectively. The thickness of the sandwiched elastic layer is taken as H = 0.0004 m. It is observed that the characteristic length of the material significantly affects the phase velocity profiles. As the characteristic length parameter l increases, the phase velocity profiles of the Love wave increases.

5.2. Effect of fluid viscosity

The effect of the fluid viscosity is depicted in Fig. 3. The dispersion curves are plotted for different values of the coefficient of the viscosity parameter η_1 . Here, characteristic length of the material is fixed, i.e. l = 0.0001 m, and thickness of the elastic layer is taken as H = 0.0004 m. The density of the viscous liquid is also fixed i.e. $\rho_1 = 1000 \text{ kg/m}^3$. It is observed that the viscosity parameter disfavors the phase velocity. As the viscosity of the material is increasing, the phase velocity of the Love waves decreases.



Fig. 3. Variation of the dimensionless phase velocity c/c_1 against the dimensionless wave number kH for different values of the coefficient of viscosity η_1

5.3. Effect of thickness of sandwiched elastic layer

The effect of thickness of the sandwiched elastic layer on the Love wave propagation in a hybrid structure consisting of an elastic layer in the middle overlying a semi infinite size dependent couple stress substrate loaded with a viscous liquid (Newtonian) half space is shown in Fig. 4. To capture this effect, we have considered different values of thickness H = 0.0006 m, 0.0008 m, 0.001 m of the sandwiched elastic layer. Also we have considered the characteristic length of the material as l = 0.0001 m and the viscosity and density of the viscous (Newtonian) liquid as $\eta_1 = 50$ Pas and $\rho_1 = 1000$ kg/m³, respectively. It is observed that with an increase in thickness of the layer, the phase velocity decreases.



Fig. 4. Variation of the dimensionless phase velocity c/c_1 against the dimensionless wave number kl for different values of thickness of the elastic layer H



Fig. 5. Variation of the dimensionless phase velocity c/c_1 against the dimensionless wave number kH for different values of density of the viscous liquid ρ_1

5.4. Effect of density of viscous liquid loading

To study the effect of the density parameter of the viscous liquid (Newtonian) on the velocity profile, we considered density of the fluid ranging from 800 to 1000 kg/m^3 for the viscosity parameter $\eta_1 = 60 \text{ Pas}$ (Shah and Balasubramaniam, 2000). It can be seen from Fig. 5 that an increase in density of the fluid leads to a decrease in the phase velocity.

6. Conclusions and applications

The analytical expression for the dispersion relation is obtained and results are graphically displayed for propagation of the Love type wave in an elastic layer sandwiched between a couple stress half space and a viscous liquid (Newtonian). The study involves a number of parameters having significant effects on the velocity profile of the Love waves. It is observed that the velocity profiles are significantly affected by variation in the associated parameters. The following conclusions are observed in the present study:

- The graphical representation reveals that the phase velocity increases with an increase in the characteristic length parameter.
- The viscosity parameter has a reverse effect on the phase velocity. As the viscosity parameter of the viscous liquid (Newtonian) increases, the phase velocity of the wave decreases.
- Thickness of the sandwiched elastic layer also disfavors the velocity profile of the wave. An increase in thickness of the elastic layer leads to a decrease in phase velocity of the wave.
- Density of the viscous liquid (Newtonian) is also unfavorable to the velocity profile of the Love wave. An increase in density of the viscous liquid (Newtonian) decreases the phase velocity profile of propagation of the Love wave.

This study can be implemented in optimization of SAW devices and other liquid sensors. The growing needs of microelectronics and other fields of modern technology require introduction and development of methods of nondestructive analysis of multilayered materials. The basis of these methods lies in acoustic waves, thermal or electromagnetic waves. The present analysis considering microstructural properties of the substrate may provide more efficient applications of the Love wave based devices designed for similar environments.

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